GO Aperture Field of Omnidirectional Axis-Displaced Dual-Reflector Antennas

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Introduction

Dual-reflector antennas have been investigated for omnidirectional coverage, where the reflector surfaces are bodies of revolution obtained by spinning confocal conic sections or shaped generating curves about a common symmetry axis [1]–[3]. Close-form design equations have been derived for omnidirectional arrangements generated by axis displaced conic sections [4]–[6]. The geometrical optics (GO) aperture field was also derived but only for broadside-pattern geometries [4]. The aim of this work is to complement the previous investigations by deriving the GO aperture field of omnidirectional axis-displaced dual-reflector antennas with an arbitrary main-beam direction in the elevation plane. GO principles [7] are employed to obtain the vector field at the antenna conical aperture. The radiation pattern is then obtained by the aperture field method (ApM). Case studies are analyzed by the accurate moment-method (MoM) technique to validate the GO-ApM analysis procedure.

Vector GO Aperture Field

Figures 1 and 2 show the omnidirectional axis-displaced ellipse (OADE) and Cassegrain (OADC) configurations, respectively, whose geometries are fully discussed in [5] and [6]. Figure 3 illustrates the most relevant geometrical parameters for an OADC configuration (the OADE parameters are defined likewise). The GO aperture field of these configurations will be derived from the relation between the input feed-ray direction θ_F and the position of the corresponding aperture point A (with the help of the auxiliary Cartesian system x_M, z_M illustrated in Figs. 1, 2, and 3). The feed ray, which intercepts the subreflector at S, makes an angle θ_F with respect to the z-axis and, after the reflection at S, an angle θ_M with respect to the z_M-axis. From Fig. 3 one readily observes that $\theta_F - \theta_M = \gamma$ and, consequently,

$$\eta(\theta_M) = [1 + \eta(\gamma)\,\eta(\theta_F)]/[\eta(\gamma) - \eta(\theta_F)]\ , \tag{1}$$

where $\eta(\alpha) = \cot(\alpha/2)$. Following the procedure in [7], let x_{MA} be the x_M-coordinate of the aperture point A (see Fig. 3) and ℓ_o the constant optical path length from the source (at O) to the x_M-axis. Then, one can show that [7]:

$$\eta(\theta_M) = [c_1 - c_2(x_{MA}/\ell_o)]/[c_2 + d_2(x_{MA}/\ell_o)]\ , \tag{2}$$

where

$$c_1 = 1 + e\,\cos(\beta-\gamma) - [2c\,(1-e^2)]/(e\,\ell_o) \qquad c_2 = e\,\sin(\beta-\gamma)$$
$$\ell_o = |\vec{Q}| + |\vec{P}_i - \vec{Q}| - \hat{z}_M\cdot\vec{P}_i \qquad d_2 = e\,\cos(\beta-\gamma) - 1 \tag{3}$$

F is the parabola focal length, $2c$ and e are the subreflector interfocal distance and eccentricity, respectively, β is the axial tilt, and the vectors $\vec{Q}$, $\vec{P}_1$, and $\vec{P}_2$ locate Q, P_1, and P_2, respectively, with $\vec{P}_i = \vec{P}_1$ or $\vec{P}_2$ for the OADE or OADC, respectively. So, from (1) and (2) one obtains the correspondence between θ_F and x_{MA}:

$$x_{MA} = \ell_o \, [c_1 - c_2 \, \eta(\theta_F - \gamma)]/[c_2 + d_2 \, \eta(\theta_F - \gamma)] \, . \tag{4}$$

The z_M-coordinate of A (i.e., z_{MA}) defines the aperture location (see Fig. 3) and can be any positive constant, in principle.

To obtain the GO aperture field, we apply the conservation of energy to the bundle of rays that leaves the feed phase center (at O) with an infinitesimal solid angle $d\Omega = \sin\theta_F \; d\theta_F \; d\phi_F$ and crosses the aperture through an infinitesimal area $ds = \rho_A \; dx_{MA} \; d\phi_F$, where

$$\rho_A = x_{MA} \, \cos\gamma + z_{MA} \, \sin\gamma \tag{5}$$

is the distance from the aperture point A to the symmetry axis (see Fig. 3). So, let the spherical TEM field radiated by the feed be described as

$$\vec{E}_F = \left[E_\theta(\theta_F) \, \hat{\theta}_F + E_\phi(\theta_F) \, \hat{\phi}_F \right] e^{-jk_o r_F}/r_F \; , \tag{6}$$

$$\vec{H}_F = (\hat{r}_F \times \vec{E}_F)/Z_o \; , \tag{7}$$

where $k_o = 2\pi/\lambda_o$, $Z_o = \sqrt{\mu_o/\varepsilon_o}$, and ($r_F$,$\theta_F$,$\phi_F$) are the usual spherical coordinates with respect to the (x,y,z) Cartesian coordinates (note that O is at the origin), while $E_\theta(\theta_F)$ and $E_\phi(\theta_F)$ define the omnidirectional feed radiation. Using GO principles and with the help of Figs. 1, 2, and 3, one observes that, after the two reflections, the vector GO field at the antenna conical aperture can be written as

$$\vec{E}_A = A_{GO} \left[E_\theta(\theta_F) \, \hat{\rho}_M + E_\phi(\theta_F) \, \hat{\phi}_M \right] e^{-jk_o(\ell_o + z_{MA})} \; , \tag{8}$$

$$\vec{H}_A = (A_{GO}/Z_o) \left[E_\theta(\theta_F) \, \hat{\phi}_M - E_\phi(\theta_F) \, \hat{\rho}_M \right] e^{-jk_o(\ell_o + z_{MA})} \; , \tag{9}$$

where $\ell_o + z_{MA}$ is the constant optical path length from O to A (see Fig. 3), A_{GO} is the GO attenuation factor (to be determined from the conservation of energy), and

$$\hat{\rho}_M = \cos\gamma \, (\cos\phi_F \, \hat{x} + \sin\phi_F \, \hat{y}) - \sin\gamma \, \hat{z} \; , \tag{10}$$

$$\hat{\phi}_M = \hat{\phi}_F = \cos\phi_F \, \hat{y} - \sin\phi_F \, \hat{x} \; . \tag{11}$$

Note that $\hat{\rho}_M = \hat{x}_M$ in the plane $\phi_F = 0$ of Fig. 3. The conservation of energy along the ray tube is further applied to obtain the amplitude A_{GO}:

$$|\vec{E}_F|^2 \, r_F^2 \; \sin\theta_F \; d\theta_F \; d\phi_F = |\vec{E}_A|^2 \; \rho_A \; dx_{MA} \; d\phi_F \; , \tag{12}$$

which implies, with the help of (6) and (8), that

$$|A_{GO}|^2 = \left(\frac{\sin\theta_F}{\rho_A} \right) \frac{d\theta_F}{dx_{MA}} = \left[\frac{2\,\eta(\theta_F)}{\rho_A \, [1 + \eta^2(\theta_F)]} \right] \frac{d\theta_F}{dx_{MA}} \; . \tag{13}$$

From (1)–(3) and the parabola's polar equation, one can rewrite (13) as

$$|A_{GO}|^2 = \left\{ \frac{2\,\eta(\theta_F) \, [\eta(\gamma) - \eta(\theta_F)]^2}{\rho_A \, F \, (e^2 - 1) \, [1 + \eta^2(\gamma)]} \right\} \left[\frac{c_2 + d_2 \, \eta(\theta_F - \gamma)}{1 + \eta^2(\theta_F)} \right]^2 . \tag{14}$$

Case Studies

To illustrate the usefulness of the GO aperture field, the radiation patterns of two representative antennas were calculated by the ApM and the results compared with MoM analyses. For the GO-ApM scheme, the adopted feed model was

$$E_\theta(\theta_F) = [J_0(kR_i \sin\theta_F) - J_0(kR_e \sin\theta_F)]/\sin\theta_F \qquad \text{and} \qquad E_\phi(\theta_F) = 0\,, \quad (15)$$

which approximately corresponds to the radiation of a TEM coaxial horn with internal and external radii R_i and R_e, respectively [3]. Here, $R_i = 0.38\lambda$ and $R_e = 1.1\lambda$. The MoM analyses accounted for the actual coaxial horn structure, which is depicted together with the dual-reflector geometries in Figs. 4(a) and 5(a) for the adopted OADE and OADG configurations, respectively. Both reflector arrangements were designed with the help of [6] and have their omnidirectional main beams at $\theta \approx 45°$. Their principal dimensions are described in the scale view of Figs. 4(a) and 5(a). The corresponding radiation patterns are illustrated in Figs. 4(b) and 5(b), where the dashed lines correspond to the GO-ApM analyses. For ignoring diffraction and coupling effects, the GO-ApM analysis can not predict the behavior of the side lobes. However, it satisfactorily provides the electrical characteristics of the main beam. Table I lists the gains and half-power beam widths (HPBW) provided by both analysis for the OADE and OADC antennas.

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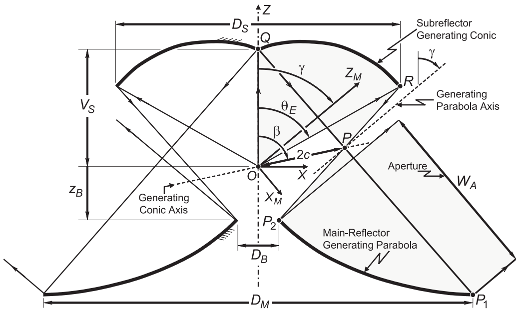

Fig. 1: OADE configuration.

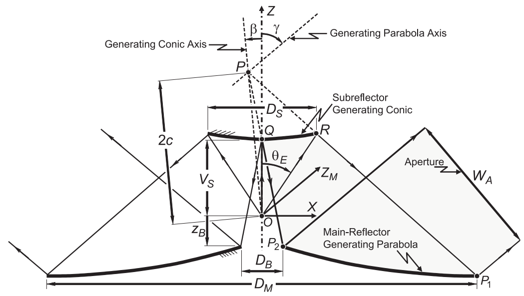

Fig. 2: OADC configuration.

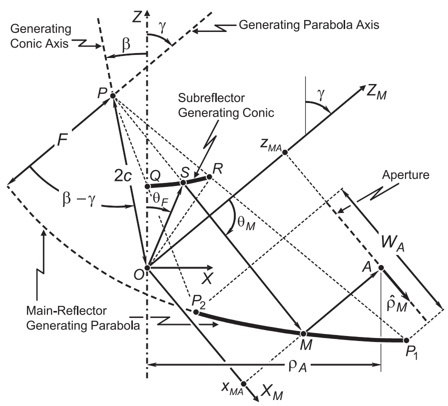

Fig. 3: Geometrical parameters.

Analysis Method	Gain (dBi)	
	OADE	OADC
GO-ApM	13.51	13.55
MoM	13.44	13.30
Analysis Method	**HPBW (deg.)**	
	OADE	OADC
GO-ApM	6.6	6.6
MoM	6.5	6.8

Table I: Antenna gains and HPBW.

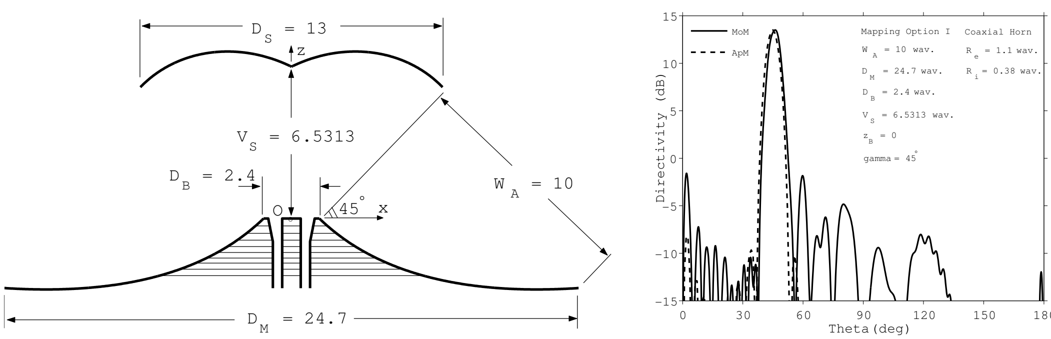

Fig. 4: OADE case study: (a) geometry and (b) radiation pattern.

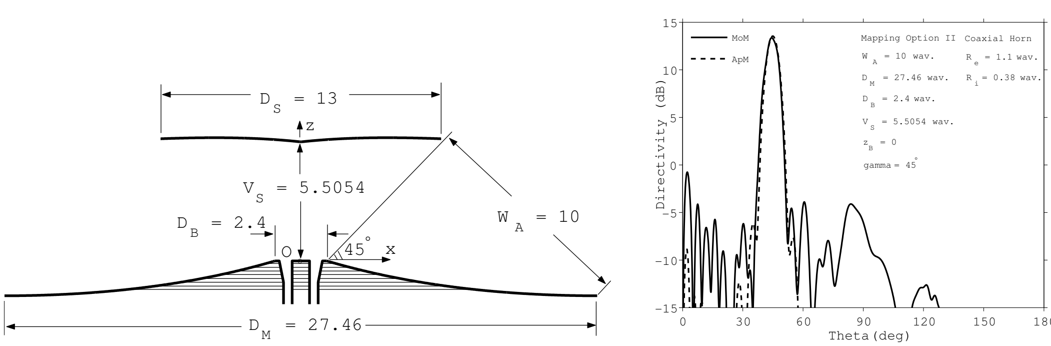

Fig. 5: OADC case study: (a) geometry and (b) radiation pattern.